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Review article

Estimation of scale parameter of inverse gaussian distribution under a bayesian framework using different loss functions

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ABSTRACT

In this paper, the Bayesian analysis of scale parameter of inverse Gaussian distribution has been considered. The Bayes estimators along with corresponding risks have been derived under a class of priors and using various loss functions. The Bayesian credible intervals have been derived for the said parameter. In order to predict the future values of the variable the posterior predictive distributions have been constructed under different priors. A simulation study has been conducted for different parametric values to assess and compare the performance of different estimators.

1. Introduction

The two-parameter inverse Gaussian (IG) distribution, as a first passage time distribution in Brownian motion, found a variety of applications in the life testing, reliability and financial modeling problems. It has statistical properties analogous to normal distribution. Gupta and Akman (1995) considered the Bayes estimation in a mixture inverse Gaussian model. Aase (2000) showed that IG distribution fits the economic indices, remarkably well in empirical investigations. Nadarajah and Kotz (2007) gave the distribution of ratio of two economic indices each having IG distribution, for comparing the consumer price indices of six major economies. Dey (2010) obtained the Bayes estimators and associated risks of the shape parameter of generalized exponential distribution, based on a class of non-informative priors using different loss functions. Pandey and Kumarrao (2010) estimated the scale parameter of the inverse Gaussian distribution under linex loss function. Prakash (2011) studied the properties of the Bayes Shrinkage estimator for the measure of dispersion of an inverse Gaussian model under the Minimax estimation criteria.

2. Model and likelihood function

The probability density function of the inverse Gaussian distribution is:

$$f(x;\lambda)=\left(\frac{\lambda}{2\pi x^3}\right)^{\frac{1}{2}}\exp\left[\frac{-\lambda(x-\mu)^2}{2\mu^2 x}\right] \quad \text{Where; } \lambda>0,\ \mu>0,\ x>0 \tag{2.1}$$

The likelihood function for the random sample of size 'n' is:

$$L(\lambda|\underline{x})=\left(\frac{\lambda}{2\pi}\right)^{\frac{n}{2}}\prod_{i=1}^{n}\left(\frac{1}{x_i^{\frac{3}{2}}}\right)\exp\left[-\lambda\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\right] \quad \text{Where; } \underline{x}=x_1,x_2,\dots,x_n \tag{2.2}$$

3. The posterior distributions for λ under the assumption of different priors

The posterior distributions under the assumption of uniform, Jeffreys, exponential, gamma and chi square priors are presented in this section.

The uniform prior is assumed to be: $p(\lambda)\propto 1$ (3.1)

The posterior distribution under the assumption of uniform prior is:

$$p(\lambda|\underline{x})\propto\lambda^{\frac{n}{2}}\exp\left[-\lambda\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\right] \quad ; \quad \lambda>0 \tag{3.2}$$

The Jeffreys prior is defined as:

$$p_j\propto\sqrt{|I(\lambda)|}\ ; \quad \text{Here; } p_j\propto\sqrt{|I(\lambda)|}=\frac{1}{\lambda} \tag{3.3}$$

The posterior distribution under Jeffreys prior is:

$$p(\lambda|\underline{x})\propto\lambda^{\frac{n}{2}-1}\exp\left[-\lambda\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\right] \quad ; \quad \lambda>0 \tag{3.4}$$

The exponential prior is assumed to be: $p(\lambda)\propto\exp(-k\lambda)\,;\lambda>0,\ k>0$ (3.5)

Where k is a hyper-parameter.

The posterior distribution under the assumption of exponential prior is:

$$p(\lambda|\underline{x})\propto\lambda^{\frac{n}{2}}\exp\left[-\lambda\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+k\right\}\right] \quad ; \quad \lambda>0 \tag{3.6}$$

The gamma prior is assumed to be:

$$p(\lambda)\propto\lambda^{a-1}e^{-\lambda b} \quad ;\lambda>0,\ a,b>0 \tag{3.7}$$

Where; a and b are hyper-parameters.

The posterior distribution under the assumption of gamma prior is:

$$p(\lambda|\underline{x})\propto\lambda^{\frac{n}{2}+a-1}\exp\left[-\lambda\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+b\right\}\right] \quad ; \quad \lambda>0 \tag{3.8}$$

The chi square prior is assumed to be:

$$p(\lambda)\propto\lambda^{\frac{h}{2}-1}e^{-\frac{\lambda}{2}} \quad ;\ \lambda>0;\ h\in\square \tag{3.9}$$

Where; h is the hyper-parameter.

The posterior distribution under chi square prior is:

$$p\left(\lambda|\underline{x}\right)\propto\lambda^{\frac{n}{2}+\frac{h}{2}-1}\exp\left[-\lambda\left\{\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}+\frac{1}{2}\right\}\right]\quad;\qquad\lambda>0\qquad(3.10)$$

4. Bayesian estimation under the assumption of different priors and loss functions

The scale parameter of inverse Gaussian distribution has been estimated under the assumption of different priors and various loss functions including squared error loss function (SELF), quadratic loss function (QLF), entropy loss function (ELF), weighted loss function (WLF), squared logarithmic loss function (SLLF), LINEX loss function (LLF), precautionary loss function (PLF) and weighted balanced loss function (WBLF). The results are given in the following tables.

Table 4.1

Bayes estimation under uniform and Jeffreys priors for different loss functions.

Loss Function		Uniform prior	Jeffreys prior
SELF	Estimator	$\left(\frac{n}{2}+1\right)\left\{\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right\}^{-1}$	$\left(\frac{n}{2}\right)\left\{\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right\}^{-1}$
	Risk	$\left(\frac{n}{2}+1\right)\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right]^{-2}$	$\left(\frac{n}{2}\right)\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right]^{-2}$
QLF	Estimator	$\left(\frac{n}{2}-1\right)\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right]^{-1}$	$\left(\frac{n}{2}-2\right)\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right]^{-1}$
	Risk	$2n^{-1}$	$2\left(n-2\right)^{-1}$
ELF	Estimator	$\frac{n}{2}\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right]^{-1}$	$\left(\frac{n}{2}-1\right)\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right]^{-1}$
	Risk	$\sum_{k=1}^{\frac{n}{2}}\frac{1}{k}-\gamma-\ln\left(\frac{n}{2}\right)$	$\sum_{k=1}^{\frac{n}{2}-1}\frac{1}{k}-\gamma-\ln\left(\frac{n}{2}-1\right)$
WLF	Estimator	$\frac{n}{2}\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right]^{-1}$	$\left(\frac{n}{2}-1\right)\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right]^{-1}$
	Risk	$\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right]^{-1}$	$\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right]^{-1}$
SLLF	Estimator	$\exp\left\{E\left(\ln\lambda\right)\right\}$	$\exp\left\{E\left(\ln\lambda\right)\right\}$
	Risk	$E\left(\ln\lambda\right)^2-\left\{E\left(\ln\lambda\right)\right\}^2$	$E\left(\ln\lambda\right)^2-\left\{E\left(\ln\lambda\right)\right\}^2$

LLF	Estimator	$-\left(\frac{n}{2}+1\right)\ln\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+1\right\}^{-1}\right]$	$-\left(\frac{n}{2}\right)\ln\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+1\right\}^{-1}\right]$
	Risk	$\lambda_{SELF}-\lambda_{LLF}$	$\lambda_{SELF}-\lambda_{LLF}$
PLF	Estimator	$\left[\left(\frac{n}{2}+2\right)\left(\frac{n}{2}+1\right)\right]^{\frac{1}{2}}\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\right]^{-1}$	$\left[\left(\frac{n}{2}+1\right)\left(\frac{n}{2}\right)\right]^{\frac{1}{2}}\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\right]^{-1}$
	Risk	$2\left(\lambda_{PLF}-\lambda_{SELF}\right)$	$2\left(\lambda_{PLF}-\lambda_{SELF}\right)$
WBLF	Estimator	$\left(\frac{n}{2}+2\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\right]^{-1}$	$\left(\frac{n}{2}+1\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\right]^{-1}$
	Risk	$2(n+4)^{-1}$	$2(n+2)^{-1}$

Table 4.2
Bayes estimation under exponential and gamma prior for different loss functions.

Loss Function		Exponential prior	Gamma prior
SELF	Estimator	$\left(\frac{n}{2}+1\right)\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+k\right\}^{-1}$	$\left(\frac{n}{2}+a\right)\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+b\right\}^{-1}$
	Risk	$\left(\frac{n}{2}+1\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+k\right]^{-2}$	$\left(\frac{n}{2}+a\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+b\right]^{-2}$
QLF	Estimator	$\left(\frac{n}{2}-1\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+k\right]^{-1}$	$\left(\frac{n}{2}+a-2\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+b\right]^{-1}$
	Risk	$2n^{-1}$	$2(n+a-2)^{-1}$
ELF	Estimator	$\frac{n}{2}\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+k\right]^{-1}$	$\left(\frac{n}{2}+a-1\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+b\right]^{-1}$
	Risk	$\sum_{k=1}^{\frac{n}{2}}\frac{1}{k}-\gamma-\ln\left(\frac{n}{2}\right)$	$\sum_{k=1}^{\frac{n}{2}+a-1}\frac{1}{k}-\gamma-\ln\left(\frac{n}{2}+a-1\right)$
WLF	Estimator	$\frac{n}{2}\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+k\right]^{-1}$	$\left(\frac{n}{2}+a-1\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+b\right]^{-1}$
	Risk	$\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+k\right]^{-1}$	$\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+b\right]^{-1}$
SLLF	Estimator	$\exp\{\mathrm{E}(\ln\lambda)\}$	$\exp\{\mathrm{E}(\ln\lambda)\}$
	Risk	$\mathrm{E}(\ln\lambda)^2-\{\mathrm{E}(\ln\lambda)\}^2$	$\mathrm{E}(\ln\lambda)^2-\{\mathrm{E}(\ln\lambda)\}^2$

LLF	Estimator	$-\left(\frac{n}{2}+1\right)\ln\left[\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+k\right\}\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+k+1\right\}^{-1}\right]$	$-\left(\frac{n}{2}+a\right)\ln\left[\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+b\right\}\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+b+1\right\}^{-1}\right]$
	Risk	$\lambda_{SELF}-\lambda_{LLF}$	$\lambda_{SELF}-\lambda_{LLF}$
PLF	Estimator	$\left[\left(\frac{n}{2}+2\right)\left(\frac{n}{2}+1\right)\right]^{\frac{1}{2}}\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+k\right]^{-1}$	$\left[\left(\frac{n}{2}+a+1\right)\left(\frac{n}{2}+a\right)\right]^{\frac{1}{2}}\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+b\right]^{-1}$
	Risk	$2\left(\lambda_{PLF}-\lambda_{SELF}\right)$	$2\left(\lambda_{PLF}-\lambda_{SELF}\right)$
WBLF	Estimator	$\left(\frac{n}{2}+2\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+k\right]^{-1}$	$\left(\frac{n}{2}+a+1\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+b\right]^{-1}$
	Risk	$2(n+4)^{-1}$	$2(n+2a+4)^{-1}$

Table 4.3
Bayes estimation under chi square prior for different loss functions.

Loss Functions	Estimator	Risk
SELF	$\left(\frac{n}{2}+\frac{h}{2}\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+\frac{1}{2}\right]^{-1}$	$\left(\frac{n}{2}+\frac{h}{2}\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+\frac{1}{2}\right]^{-2}$
QLF	$\left(\frac{n}{2}+\frac{h}{2}-2\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+\frac{1}{2}\right]^{-1}$	$2(n+h-2)^{-1}$
ELF	$\left(\frac{n}{2}+\frac{h}{2}-1\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+\frac{1}{2}\right]^{-1}$	$\sum_{k=1}^{\frac{n}{2}+\frac{h}{2}-1}\frac{1}{k}-\gamma-\ln\left(\frac{n}{2}+\frac{h}{2}-1\right)$
WLF	$\left(\frac{n}{2}+\frac{h}{2}-1\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+\frac{1}{2}\right]^{-1}$	$\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+\frac{1}{2}\right]^{-1}$
SLLF	$\exp\{\mathrm{E}(\ln\lambda)\}$	$\mathrm{E}(\ln\lambda)^2-\{\mathrm{E}(\ln\lambda)\}^2$
LLF	$-\left(\frac{n}{2}+\frac{h}{2}\right)\ln\left[\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+\frac{1}{2}\right\}\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+\frac{1}{2}+1\right\}^{-1}\right]$	$\lambda_{SELF}-\lambda_{LLF}$
PLF	$\left[\left(\frac{n}{2}+\frac{h}{2}\right)\left(\frac{n}{2}+\frac{h}{2}+1\right)\right]^{\frac{1}{2}}\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+\frac{1}{2}\right]^{-1}$	$2\left(\lambda_{PLF}-\lambda_{SELF}\right)$
WLF	$\left(\frac{n}{2}+\frac{h}{2}+1\right)\left[\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2x_i}+\frac{1}{2}\right]^{-1}$	$\frac{2}{n+h+4}$

Where $\gamma=0.57721$ is an Euler constant.

5. Posterior predictive distributions under different priors

The posterior predictive distribution is defined to be:

$$p\left(y|x\right)=\int_0^\infty p(\lambda|\underline{x})\,f\left(y;\lambda\right)d\lambda \quad (5.1)$$

The posterior predictive distributions under the assumption of uniform, Jeffreys, exponential, gamma and chi square priors are presented in the following respectively.

$$p_U\left(y|x\right)=\frac{\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right]^{\frac{n}{2}+1}\Gamma\left(\frac{n}{2}+\frac{3}{2}\right)}{\Gamma\left(\frac{n}{2}+1\right)\sqrt{2\pi}y^{\frac{3}{2}}\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}+\frac{\left(y-\mu\right)^2}{2\mu^2y}\right]^{\frac{n}{2}+\frac{3}{2}}} \quad ; \quad y>0 \quad (5.2)$$

$$p_J\left(y|x\right)=\frac{\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}\right]^{\frac{n}{2}}\Gamma\left(\frac{n}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\sqrt{2\pi}y^{\frac{3}{2}}\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}+\frac{\left(y-\mu\right)^2}{2\mu^2y}\right]^{\frac{n}{2}+\frac{1}{2}}} \quad ; \quad y>0 \quad (5.3)$$

$$p_E\left(y|x\right)=\frac{\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}+k\right]^{\frac{n}{2}+1}\Gamma\left(\frac{n}{2}+\frac{3}{2}\right)}{\Gamma\left(\frac{n}{2}+1\right)\sqrt{2\pi}y^{\frac{3}{2}}\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}+k+\frac{\left(y-\mu\right)^2}{2\mu^2y}\right]^{\frac{n}{2}+\frac{3}{2}}} \quad ; \quad y>0 \quad (5.4)$$

$$p_G\left(y|x\right)=\frac{\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}+b\right]^{\frac{n}{2}+a}\Gamma\left(\frac{n}{2}+a+\frac{1}{2}\right)}{\Gamma\left(\frac{n}{2}+a\right)\sqrt{2\pi}y^{\frac{3}{2}}\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}+b+\frac{\left(y-\mu\right)^2}{2\mu^2y}\right]^{\frac{n}{2}+a+\frac{1}{2}}} \quad ; \quad y>0 \quad (5.5)$$

$$p_C\left(y|x\right)=\frac{\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}+\frac{1}{2}\right]^{\frac{n}{2}+\frac{h}{2}}\Gamma\left(\frac{n}{2}+\frac{h}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{n}{2}+\frac{h}{2}\right)\sqrt{2\pi}y^{\frac{3}{2}}\left[\sum_{i=1}^{n}\frac{\left(x_i-\mu\right)^2}{2\mu^2x_i}+\frac{1}{2}+\frac{\left(y-\mu\right)^2}{2\mu^2y}\right]^{\frac{n}{2}+\frac{h}{2}+\frac{1}{2}}} \quad ; \quad y>0 \quad (5.6)$$

6. The credible intervals under different priors

As presented by Tahir and Saleem (2011), the credible intervals using the posterior distributions under the assumption of uniform, Jeffreys, exponential, gamma and chi square priors are given in the following respectively.

$$\frac{2\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\right\}}{\chi^2_{2\left(\frac{n}{2}+1\right),\left(\frac{\alpha}{2}\right)}} < \lambda_U < \frac{2\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\right\}}{\chi^2_{2\left(\frac{n}{2}+1\right),\left(1-\frac{\alpha}{2}\right)}} \tag{6.1}$$

$$\frac{2\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\right\}}{\chi^2_{2\left(\frac{n}{2}\right),\left(\frac{\alpha}{2}\right)}} < \lambda_J < \frac{2\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}\right\}}{\chi^2_{2\left(\frac{n}{2}\right),\left(1-\frac{\alpha}{2}\right)}} \tag{6.2}$$

$$\frac{2\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+k\right\}}{\chi^2_{2\left(\frac{n}{2}+1\right),\left(\frac{\alpha}{2}\right)}} < \lambda_E < \frac{2\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+k\right\}}{\chi^2_{2\left(\frac{n}{2}+1\right),\left(1-\frac{\alpha}{2}\right)}} \tag{6.3}$$

$$\frac{2\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+b\right\}}{\chi^2_{2\left(\frac{n}{2}+a\right),\left(\frac{\alpha}{2}\right)}} < \lambda_G < \frac{2\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+b\right\}}{\chi^2_{2\left(\frac{n}{2}+a\right),\left(1-\frac{\alpha}{2}\right)}} \tag{6.4}$$

$$\frac{2\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+\frac{1}{2}\right\}}{\chi^2_{2\left(\frac{n}{2}+\frac{h}{2}\right),\left(\frac{\alpha}{2}\right)}} < \lambda_G < \frac{2\left\{\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\mu^2 x_i}+\frac{1}{2}\right\}}{\chi^2_{2\left(\frac{n}{2}+\frac{h}{2}\right),\left(1-\frac{\alpha}{2}\right)}} \tag{6.5}$$

7. Simulation study

A simulation study has been conducted to evaluate the behavior and performance of different estimators. A comparison in terms of magnitude of posterior risks is needed to check whether an estimator is inadmissible under some loss function or prior distribution. The samples have been simulated for n = 50, 100, 200, 300 and 400 using $\lambda \in (5,10)$ under 1000 replications. The risks associated with Bayes estimators are given in parenthesis.

The simulation study was conducted for four priors and under eight loss functions. From the study it can be seen that Bayes estimate of the parameter converges to the true value of λ by increasing the sample size. Using LLF the estimated value of the parameter is always less than the estimates obtained under other loss functions. The Bayes estimates obtained under ELF and WLF are same for each prior. The risk associated with estimates under ELF is the minimum for each estimate, while, the maximum risk is associated with estimates under SELF for each prior. The risks for estimates under QLF, SLLF and WBLF are similar for each prior especially for large samples.

Table 4
Bayes estimation under uniform prior using different loss functions.

Sample Size	Estimates and Risks under Loss Functions (λ = 5)							
	SELF	QLF	ELF	WLF	SLLF	LLF	PLF	WBLF
50	5.6451	5.2109	5.4280	5.4280	5.5369	5.1087	5.7526	5.8622
	(1.2257)	(0.0400)	(0.0199)	(0.2171)	(0.0372)	(0.5364)	(0.2151)	(0.0370)
100	5.2689	5.0623	5.1656	5.1656	5.2174	5.0141	5.3203	5.3722
	(0.5443)	(0.0200)	(0.0100)	(0.1033)	(0.0193)	(0.2548)	(0.1028)	(0.0192)
200	5.1498	5.0478	5.0988	5.0988	5.1244	5.0228	5.1752	5.2008
	(0.2626)	(0.0100)	(0.0050)	(0.0510)	(0.0099)	(0.1270)	(0.0509)	(0.0098)
300	5.1240	5.0561	5.0901	5.0901	5.1071	5.0390	5.1409	5.1579
	(0.1739)	(0.0067)	(0.0033)	(0.0339)	(0.0066)	(0.0850)	(0.0339)	(0.0066)
400	5.0512	5.0009	5.0261	5.0261	5.0387	4.9888	5.0637	5.0763
	(0.1269)	(0.0050)	(0.0025)	(0.0251)	(0.0050)	(0.0624)	(0.0251)	(0.0050)

Table 5
Bayes estimation under uniform prior using different loss functions.

Sample Size	Estimates and Risks under Loss Functions (λ = 10)							
	SELF	QLF	ELF	WLF	SLLF	LLF	PLF	WBLF
50	11.5355	10.6482	11.0918	11.0918	11.3145	9.5470	11.7552	11.9792
	(5.1180)	(0.0400)	(0.0199)	(0.4437)	(0.0372)	(1.9885)	(0.4395)	(0.0370)
100	10.2691	9.8664	10.0677	10.0677	10.1686	9.3560	10.3693	10.4705
	(2.0677)	(0.0200)	(0.0100)	(0.2014)	(0.0193)	(0.9131)	(0.2004)	(0.0192)
200	10.2373	10.0346	10.1359	10.1359	10.1867	9.7511	10.2879	10.3387
	(1.0376)	(0.0100)	(0.0050)	(0.1014)	(0.0099)	(0.4862)	(0.1011)	(0.0098)
300	10.0848	9.9512	10.0180	10.0180	10.0515	9.7623	10.1181	10.1516
	(0.6735)	(0.0067)	(0.0033)	(0.0668)	(0.0066)	(0.3225)	(0.0667)	(0.0066)
400	10.0715	9.9713	10.0214	10.0214	10.0465	9.8273	10.0965	10.1216
	(0.5047)	(0.0050)	(0.0025)	(0.0501)	(0.0050)	(0.2442)	(0.0500)	(0.0050)

Table 6
Bayes estimation under Jeffreys prior using different loss functions.

Sample Size	Estimates and Risks under Loss Functions (λ = 5)							
	SELF	QLF	ELF	WLF	SLLF	LLF	PLF	WBLF
50	5.4280	4.9937	5.2109	5.2109	5.3154	4.9122	5.5355	5.6451
	(1.1785)	(0.0417)	(0.0199)	(0.2171)	(0.0387)	(0.5158)	(0.2150)	(0.0385)
100	5.1656	4.9590	5.0623	5.0623	5.1130	4.9158	5.2170	5.2689
	(0.5337)	(0.0204)	(0.0100)	(0.1033)	(0.0197)	(0.2498)	(0.1028)	(0.0196)
200	5.0988	4.9968	5.0478	5.0478	5.0731	4.9731	5.1242	5.1498
	(0.2600)	(0.0101)	(0.0050)	(0.0510)	(0.0100)	(0.1257)	(0.0509)	(0.0099)
300	5.0901	5.0222	5.0561	5.0561	5.0730	5.0056	5.1070	5.1240
	(0.1727)	(0.0067)	(0.0033)	(0.0339)	(0.0067)	(0.0845)	(0.0339)	(0.0066)
400	5.0261	4.9758	5.0009	5.0009	5.0135	4.9640	5.0386	5.0512
	(0.1263)	(0.0050)	(0.0025)	(0.0251)	(0.0050)	(0.0621)	(0.0251)	(0.0050)

Table 7
Bayes estimation under Jeffreys prior using different loss functions.

Sample Size	Estimates and Risks under Loss Functions (λ = 10)							
	SELF	**QLF**	**ELF**	**WLF**	**SLLF**	**LLF**	**PLF**	**WBLF**
50	11.0918	10.2045	10.6482	10.6482	10.8619	9.1798	11.3115	11.5355
	(4.9211)	(0.0417)	(0.0199)	(0.4437)	(0.0387)	(1.9121)	(0.4393)	(0.0385)
100	10.0677	9.6650	9.8664	9.8664	9.9653	9.1725	10.1679	10.2691
	(2.0272)	(0.0204)	(0.0100)	(0.2014)	(0.0197)	(0.8952)	(0.2004)	(0.0196)
200	10.1359	9.9332	10.0346	10.0346	10.0849	9.6545	10.1865	10.2373
	(1.0274)	(0.0101)	(0.0050)	(0.1014)	(0.0100)	(0.4814)	(0.1011)	(0.0099)
300	10.0180	9.8844	9.9512	9.9512	9.9845	9.6977	10.0514	10.0848
	(0.6691)	(0.0067)	(0.0033)	(0.0668)	(0.0067)	(0.3203)	(0.0667)	(0.0066)
400	10.0214	9.9212	9.9713	9.9713	9.9963	9.7784	10.0464	10.0715
	(0.5021)	(0.0050)	(0.0025)	(0.0501)	(0.0050)	(0.2430)	(0.0500)	(0.0050)

Table 8
Bayes estimation under exponential prior using different loss functions.

Sample Size	Estimates and Risks under Loss Functions (λ = 5)							
	SELF	**QLF**	**ELF**	**WLF**	**SLLF**	**LLF**	**PLF**	**WBLF**
50	3.2076	2.9609	3.0842	3.0842	3.1461	3.0246	3.2687	3.3310
	(0.3957)	(0.0400)	(0.0199)	(0.1234)	(0.0372)	(0.1830)	(0.1222)	(0.0370)
100	3.8697	3.7179	3.7938	3.7938	3.8318	3.7299	3.9074	3.9455
	(0.2936)	(0.0200)	(0.0100)	(0.0759)	(0.0193)	(0.1398)	(0.0755)	(0.0192)
200	4.3699	4.2834	4.3267	4.3267	4.3484	4.2781	4.3915	4.4132
	(0.1891)	(0.0100)	(0.0050)	(0.0433)	(0.0099)	(0.0919)	(0.0432)	(0.0098)
300	4.5800	4.5194	4.5497	4.5497	4.5649	4.5120	4.5952	4.6104
	(0.1389)	(0.0067)	(0.0033)	(0.0303)	(0.0066)	(0.0681)	(0.0303)	(0.0066)
400	4.6428	4.5966	4.6197	4.6197	4.6313	4.5900	4.6544	4.6659
	(0.1072)	(0.0050)	(0.0025)	(0.0231)	(0.0050)	(0.0528)	(0.0231)	(0.0050)

Table 9
Bayes estimation under exponential prior using different loss functions.

Sample Size	Estimates and Risks under Loss Functions (λ = 10)							
	SELF	**QLF**	**ELF**	**WLF**	**SLLF**	**LLF**	**PLF**	**WBLF**
50	4.5187	4.1711	4.3449	4.3449	4.4321	4.1663	4.6047	4.6925
	(0.7853)	(0.0400)	(0.0199)	(0.1738)	(0.0372)	(0.3524)	(0.1722)	(0.0370)
100	6.0238	5.7876	5.9057	5.9057	5.9649	5.6938	6.0826	6.1420
	(0.7115)	(0.0200)	(0.0100)	(0.1181)	(0.0193)	(0.3300)	(0.1175)	(0.0192)
200	7.5566	7.4069	7.4817	7.4817	7.5192	7.2872	7.5939	7.6314
	(0.5654)	(0.0100)	(0.0050)	(0.0748)	(0.0099)	(0.2693)	(0.0746)	(0.0098)
300	8.1741	8.0658	8.1199	8.1199	8.1471	7.9605	8.2011	8.2282
	(0.4425)	(0.0067)	(0.0033)	(0.0541)	(0.0066)	(0.2136)	(0.0540)	(0.0066)
400	8.5688	8.4835	8.5261	8.5261	8.5475	8.3911	8.5900	8.6114
	(0.3653)	(0.0050)	(0.0025)	(0.0426)	(0.0050)	(0.1776)	(0.0426)	(0.0050)

Table 10
Bayes estimation under gamma prior using different loss functions.

Sample Size	Estimates and Risks under Loss Functions (λ = 5)							
	SELF	QLF	ELF	WLF	SLLF	LLF	PLF	WBLF
50	3.9360	3.6332	3.7846	3.7846	3.8605	3.6651	4.0109	4.0873
	(0.5958)	(0.0400)	(0.0199)	(0.1514)	(0.0372)	(0.2709)	(0.1500)	(0.0370)
100	4.3666	4.1954	4.2810	4.2810	4.3239	4.1897	4.4093	4.4523
	(0.3739)	(0.0200)	(0.0100)	(0.0856)	(0.0193)	(0.1769)	(0.0852)	(0.0192)
200	4.6732	4.5807	4.6270	4.6270	4.6502	4.5683	4.6963	4.7195
	(0.2162)	(0.0100)	(0.0050)	(0.0463)	(0.0099)	(0.1049)	(0.0462)	(0.0098)
300	4.7983	4.7348	4.7666	4.7666	4.7825	4.7237	4.8142	4.8301
	(0.1525)	(0.0067)	(0.0033)	(0.0318)	(0.0066)	(0.0747)	(0.0317)	(0.0066)
400	4.8095	4.7616	4.7855	4.7855	4.7975	4.7528	4.8214	4.8334
	(0.1151)	(0.0050)	(0.0025)	(0.0239)	(0.0050)	(0.0566)	(0.0239)	(0.0050)

Table 11
Bayes estimation under gamma prior using different loss functions.

Sample Size	Estimates and Risks under Loss Functions (λ = 10)							
	SELF	QLF	ELF	WLF	SLLF	LLF	PLF	WBLF
50	6.1120	5.6419	5.8769	5.8769	5.9949	5.4895	6.2285	6.3471
	(1.4368)	(0.0400)	(0.0199)	(0.2351)	(0.0372)	(0.6225)	(0.2329)	(0.0370)
100	7.3209	7.0338	7.1774	7.1774	7.2493	6.8409	7.3923	7.4644
	(1.0509)	(0.0200)	(0.0100)	(0.1435)	(0.0193)	(0.4800)	(0.1429)	(0.0192)
200	8.5118	8.3432	8.4275	8.4275	8.4697	8.1721	8.5538	8.5961
	(0.7173)	(0.0100)	(0.0050)	(0.0843)	(0.0099)	(0.3397)	(0.0841)	(0.0098)
300	8.8965	8.7786	8.8375	8.8375	8.8671	8.6442	8.9259	8.9554
	(0.5242)	(0.0067)	(0.0033)	(0.0589)	(0.0066)	(0.2522)	(0.0588)	(0.0066)
400	9.1541	9.0630	9.1086	9.1086	9.1314	8.9518	9.1769	9.1997
	(0.4169)	(0.0050)	(0.0025)	(0.0455)	(0.0050)	(0.2023)	(0.0455)	(0.0050)

Table 12
Bayes estimation under chi square prior using different loss functions.

Sample Size	Estimates and Risks under Loss Functions (λ = 5)							
	SELF	QLF	ELF	WLF	SLLF	LLF	PLF	WBLF
50	5.0923	4.7006	4.8964	4.8964	4.9947	4.6504	5.1893	5.2881
	(0.9974)	(0.0400)	(0.0199)	(0.1959)	(0.0372)	(0.4418)	(0.1940)	(0.0370)
100	5.0101	4.8136	4.9119	4.9119	4.9611	4.7790	5.0590	5.1083
	(0.4922)	(0.0200)	(0.0100)	(0.0982)	(0.0193)	(0.2311)	(0.0978)	(0.0192)
200	5.0218	4.9223	4.9721	4.9721	4.9970	4.9009	5.0466	5.0715
	(0.2497)	(0.0100)	(0.0050)	(0.0497)	(0.0099)	(0.1209)	(0.0496)	(0.0098)
300	5.0385	4.9718	5.0051	5.0051	5.0219	4.9563	5.0552	5.0719
	(0.1681)	(0.0067)	(0.0033)	(0.0334)	(0.0066)	(0.0822)	(0.0333)	(0.0066)
400	4.9885	4.9389	4.9637	4.9637	4.9761	4.9276	5.0009	5.0133
	(0.1238)	(0.0050)	(0.0025)	(0.0248)	(0.0050)	(0.0609)	(0.0248)	(0.0050)

Table 13
Bayes estimation under chi square prior using different loss functions.

Sample Size	Estimates and Risks under Loss Functions (λ = 10)							
	SELF	QLF	ELF	WLF	SLLF	LLF	PLF	WBLF
50	9.4411	8.7149	9.0780	9.0780	9.2602	8.0542	9.6210	9.8042
	(3.4283)	(0.0400)	(0.0199)	(0.3631)	(0.0372)	(1.3869)	(0.3597)	(0.0370)
100	9.3298	8.9639	9.1469	9.1469	9.2385	8.5680	9.4208	9.5127
	(1.7068)	(0.0200)	(0.0100)	(0.1829)	(0.0193)	(0.7618)	(0.1820)	(0.0192)
200	9.7435	9.5506	9.6470	9.6470	9.6954	9.3017	9.7916	9.8400
	(0.9400)	(0.0100)	(0.0050)	(0.0965)	(0.0099)	(0.4418)	(0.0962)	(0.0098)
300	9.7589	9.6297	9.6943	9.6943	9.7267	9.4565	9.7912	9.8235
	(0.6307)	(0.0067)	(0.0033)	(0.0646)	(0.0066)	(0.3024)	(0.0645)	(0.0066)
400	9.8253	9.7276	9.7765	9.7765	9.8010	9.5927	9.8498	9.8742
	(0.4803)	(0.0050)	(0.0025)	(0.0489)	(0.0050)	(0.2326)	(0.0488)	(0.0050)

The patterns of risks are almost similar for each prior and under every loss function. It is interesting to note that the magnitude of risks under ELF, QLF and SLLF are not affected by the choice of different priors and magnitude of true parameter value. However, the performance of Bayes estimator under ELF is the best, for each prior, as the risk associated with the estimate is the minimum among other estimates. It is observed that for uniform, Jeffreys and chi square priors the parameter is over and under estimated, while in case of gamma and exponential priors the parameter is under estimated. Also, the rate of convergence is faster under non-informative priors than that under informative priors. However, the size of over or under estimation is directly proportional to the magnitude of true parameter value, while it is inversely proportional to the sample size. The risks associated with estimates under informative priors are smaller than those associated with the estimates under non-informative priors. Furthermore, it should be mentioned here that, when hyper-parameters tend to be zero, the estimation in the case of the conjugate prior tend to be the estimation in the case of the Jeffreys prior and the estimation under exponential prior tend to be the estimation under uniform prior.

Finally, it can be concluded that the ELF may be preferred for obtaining the Bayes estimates of scale parameter inverse Gaussian distribution, for each prior, as the risks associated with these estimates are the minimum. Whereas, the performance of informative priors (especially under exponential prior) is better than non-informative priors for each loss function. So, ELF estimator under informative (exponential) priors provides the most efficient point estimator of the parameter.

Table 14
Credible intervals under uniform prior.

Sample Size	Parametric Values					
	λ = 5			λ = 10		
	LL	UL	UL–LL	LL	UL	UL–LL
50	3.9770	8.6417	4.6646	8.1269	17.6588	9.5319
100	4.0764	7.0764	3.0000	7.9450	13.7920	5.8470
200	4.2764	6.3225	2.0461	8.5011	12.5686	4.0675
300	4.4229	6.0944	1.6716	8.7049	11.9947	3.2899
400	4.4403	5.8606	1.4202	8.8535	11.6853	2.8318

Table 15
Credible intervals under Jeffreys prior.

Sample Size	Parametric Values					
	λ = 5			λ = 10		
	LL	UL	UL–LL	LL	UL	UL–LL
50	3.8000	8.3876	4.5876	7.7652	17.1398	9.3746
100	3.9870	6.9596	2.9727	7.7707	13.5644	5.7937
200	4.2304	6.2667	2.0363	8.4095	12.4575	4.0480
300	4.3645	6.0140	1.6495	8.5900	11.8364	3.2464
400	4.3963	5.8024	1.4062	8.7656	11.5693	2.8037

Table 16
Credible intervals under exponential prior.

Sample Size	Parametric Values					
	λ = 5			λ = 10		
	LL	UL	UL–LL	LL	UL	UL–LL
50	2.4598	5.1103	2.6505	6.1835	9.9173	3.7338
100	2.9939	5.1972	2.2033	6.6605	10.0904	3.4299
200	3.6288	5.3651	1.7363	7.2750	10.2774	3.0023
300	3.9533	5.4474	1.4941	8.0556	10.7222	2.6666
400	4.0813	5.3868	1.3054	8.5325	10.9418	2.4093

Table 17
Credible intervals under gamma prior.

Sample Size	Parametric Values					
	λ = 5			λ = 10		
	LL	UL	UL–LL	LL	UL	UL–LL
50	2.7729	6.0253	3.2523	4.3060	9.3564	5.0504
100	3.3784	5.8647	2.4863	5.6640	9.8324	4.1684
200	3.8807	5.7375	1.8568	7.0683	10.4501	3.3819
300	4.1418	5.7071	1.5653	7.6791	10.5814	2.9022
400	4.2278	5.5801	1.3523	8.0471	10.6209	2.5739

Table 18
Credible intervals under chi square prior.

Sample Size	Parametric Values					
	λ = 5			λ = 10		
	LL	UL	UL–LL	LL	UL	UL–LL
50	3.5876	7.7954	4.2078	6.6514	14.4527	7.8013
100	3.8762	6.7289	2.8527	7.2182	12.5305	5.3122
200	4.1701	6.1654	1.9952	8.0911	11.9623	3.8713
300	4.3491	5.9927	1.6437	8.4236	11.6071	3.1836
400	4.3852	5.7878	1.4026	8.6371	11.3997	2.7626

Further, it can be indicated that for each value of the parameter the width of credible intervals under exponential is the minimum in most of the cases. The width of intervals is directly proportional to the true value of the parameter, while it is inversely proportional to the sample size for each prior. It is interesting to note that each credible interval contains the true and estimated (under each loss function) value of the parameter with some exception. Finally, it can be concluded that the credible intervals under the informative prior are narrower. A better choice of hyper parameters may result in more precise credible intervals under informative priors.

8. Bayesian analysis under real life data

In this section, the data set of repair times (in hours) of 30 air conditioning systems presented by Sinha and Prabha (2010) have been used to discuss the practical applications of the results obtained in previous sections.

Table 19
Bayesian estimation under different loss functions.

Priors	Loss Functions							
	SELF	**QLF**	**ELF**	**WLF**	**SLLF**	**LLF**	**PLF**	**WBLF**
Uniform	1.1348 (0.0805)	0.9930 (0.0667)	1.0639 (0.0330)	1.0639 (0.0709)	1.0996 (0.0606)	1.0964 (0.0384)	1.1698 (0.0699)	1.2058 (0.0588)
Jeffreys	1.0639 (0.0755)	0.9221 (0.0714)	0.9930 (0.0353)	0.9930 (0.0709)	1.0286 (0.0644)	1.0279 (0.0360)	1.0988 (0.0698)	1.1348 (0.0625)
Expo	0.9938 (0.0617)	0.8696 (0.0667)	0.9317 (0.0330)	0.9317 (0.0621)	0.9630 (0.0606)	0.9642 (0.0296)	1.0244 (0.0612)	1.0560 (0.0588)
Gamma	1.0560 (0.0656)	0.9317 (0.0667)	0.9938 (0.0325)	0.9938 (0.0621)	1.0267 (0.0572)	1.0245 (0.0315)	1.0866 (0.0612)	1.1181 (0.0556)
Chi Square	1.0960 (0.0751)	0.9590 (0.0667)	1.0275 (0.0330)	1.0275 (0.0685)	1.0619 (0.0606)	1.0601 (0.0359)	1.1297 (0.0675)	1.1645 (0.0588)

The above table shows that the risks associated with estimates under exponential prior are the minimum for each loss function. On the other hand, the risks associated with estimates under ELF are the least for every prior. So, the pattern of results using the real life data is similar to those under simulation study.

Table 20
Credible intervals under different priors.

Limits	Uniform	Jeffreys	Exponential	Gamma	Chi Square
LL	0.5037	0.5261	0.5751	0.5517	0.5215
UL	1.4325	1.4719	1.4558	1.4474	1.4109
Difference	0.9289	0.9458	0.8807	0.8958	0.8893

The credible interval under exponential prior is narrower as compared to those under other prior. The findings of credible intervals are again completely in accordance with the simulation study.

9. Conclusions and recommendations

The above analysis suggests that the performance of the estimates under uniform prior is better than those under Jeffreys prior for most of the cases. While in case of informative priors (used), the performance of estimates using exponential prior is the best in terms of Bayes risks. Similarly, in comparison of informative and non-informative priors, the informative priors give the better results. Although, the estimates under informative priors converge to the estimates under non-informative priors as the values of hyper-parameters approach to zero. It can also be indicated that the estimates under ELF are associated with the minimum risks using each prior.

So in situations involving the estimation of scale parameter of inverse Gaussian distribution using Bayesian framework, the estimator under ELF for informative (especially exponential) priors could be effectively employed.

The study can further be extended for more informative priors and considering both scale and shape parameters. Some other loss functions can also be included in the further study.

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